

THE SAGE'S LIBRARY.
EDUCATION UNTOLD

GEOMETRY

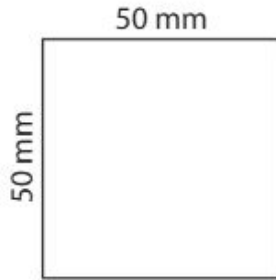
Measurement, Area and Volume of
2D & 3D Shapes



Pop Quiz: Review from Last Year

Find the area and perimeter of each square.

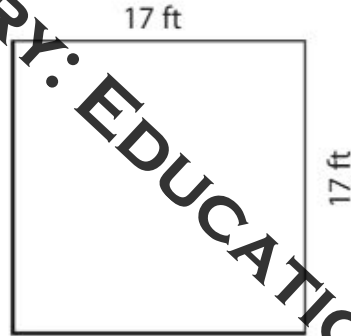
1)



Area : _____

Perimeter : _____

2)



Area : _____

Perimeter : _____

3)

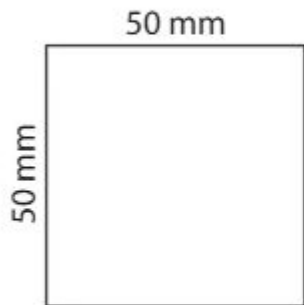


Area : _____

Perimeter : _____

Find the area and perimeter of each square.

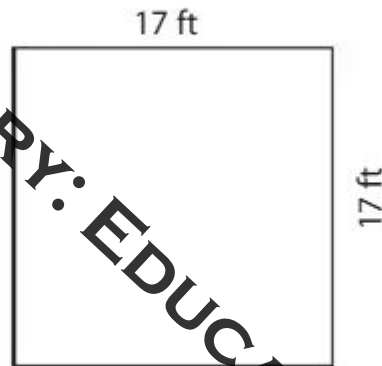
1)



Area : 2500 mm²

Perimeter : 200 mm

2)



Area : 289 ft²

Perimeter : 68 ft

3)



Area : 961 m²

Perimeter : 124 m

Measurement & Geometry

Where do you see shapes and measurements in your everyday life?



Measurement

Key Concepts:

- Measurement involves determining the size, length, area, or volume of an object or space.
- The concept of measurement is essential in daily life and is used to calculate distances, areas, volumes, and other physical quantities.

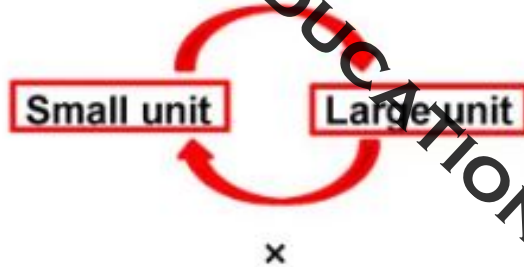
Units of Measurement:

- **Metric System:** millimeters (mm), centimeters (cm), meters (m), kilometers (km), grams (g), liters (L)
- **Imperial System:** inches (in), feet (ft), yards (yd), miles (mi), pounds (lb), gallons (gal)

Conversion of Units

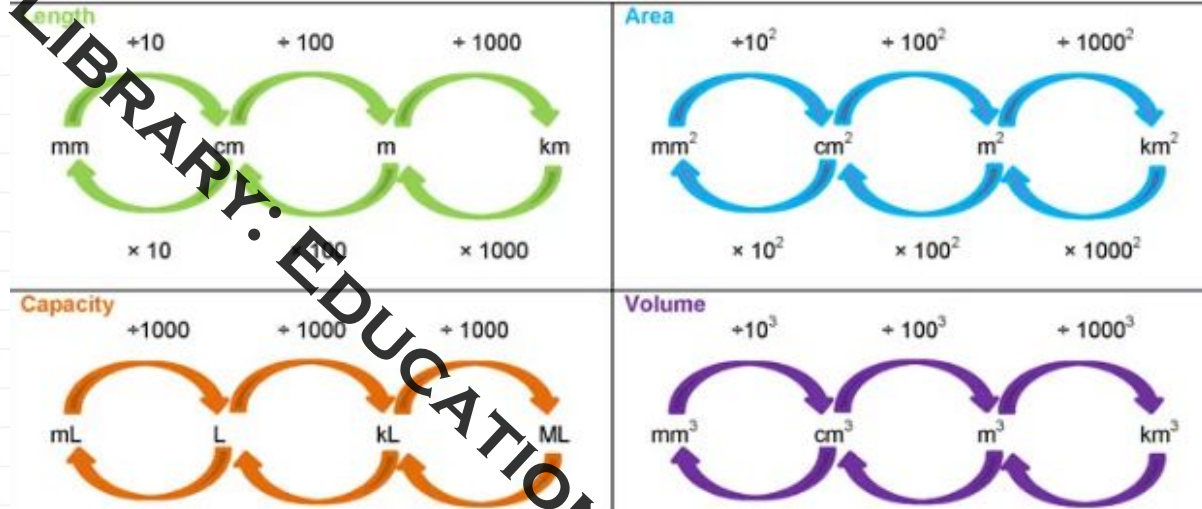
Conversion between units is essential for solving problems in real-world contexts.

- Example: Convert 2.5 meters to centimeters ($2.5 \text{ m} = 250 \text{ cm}$).
- Conversion formula: Multiply or divide by powers of 10 (e.g., 1 meter = 100 centimeters).



Unit Conversions

Depending on what you are trying to do, you will either multiply or divide to get a new unit.



How to Convert

Step 1: Identify the unknown amount and its units:

- Determine what you're trying to find. For example, if you're converting 5 kilometers to meters, the unknown amount is the number of meters.
- Example: Convert 5 kilometers to meters.
 - Unknown amount: Meters
 - Unknown units: Meters

2. Identify the known amount and its units:

- Look at the given value and its unit.
- Example: 5 kilometers.
 - Known amount: 5
 - Known units: Kilometers

By identifying the units, understanding whether you're converting to a larger or smaller unit, and applying multiplication or division, you can successfully convert between different units.

How to Convert

3. Determine which operation to use: multiply or divide:

- Use multiplication or division based on the relationship between the units.
 - If you are converting from a larger unit to a smaller unit, **multiply** by the conversion factor.
 - If you are converting from a smaller unit to a larger unit, **divide** by the conversion factor.




Example 1: Kilometers to Meters

- Known unit: kilometers (larger unit)
- Unknown unit: meters (smaller unit)
- Conversion factor: 1 kilometer = 1,000 meters.
- **Multiply:** 5 kilometers \times 1,000 = 5,000 meters.

Example 2: Meters to Kilometers

- Known unit: meters (smaller unit)
- Unknown unit: kilometers (larger unit)
- Conversion factor: 1 kilometer = 1,000 meters.
- **Divide:** 500 meters \div 1,000 = 0.5 kilometers.

Metric Conversion

King	Henry	David	Unusually	Drinking	Chocolate	Milk
Kilo	Hecto	Deca	 * Unit *	Deci	Centi	Milli
 $10 \times 10 \times 10 \times$ LARGER than a unit	$10 \times 10 \times$ LARGER than a unit	$10 \times$ LARGER than a unit	Meter <i>(length)</i> Liter <i>(liquid volume)</i> Gram <i>(mass/weight)</i> 1 unit	$10 \times$ SMALLER than a unit	$10 \times 10 \times$ SMALLER than a unit	$10 \times 10 \times 10 \times$ SMALLER than a unit 
1 kilo = 1,000 units	1 hecto = 100 units	1 deca = 10 units		10 deci = 1 unit	100 centi = 1 unit	1,000 milli = 1 unit
km = kilometer kL = kiloliter kg = kilogram	hm = hectometer hL = hectoliter hg = hectogram	dam = decameter daL = decaliter dag = decagram	m = meter L = liter g = gram	dm = decimeter dL = deciliter dg = decigram	cm = centimeter cL = centiliter cg = centigram	mm = millimeter mL = milliliter mg = milligram

Example: 5 kilo

50 hecto

500 deca

5,000 units

50,000 deci

500,000 centi

5,000,000 milli

DIVIDE numbers by 10 if you are getting bigger (same as moving decimal point one space to the left)

MULTIPLY numbers by 10 if you are getting smaller (same as moving decimal point one space to the right)

METRIC CONVERSION

- 1) Bigger to Smaller $\rightarrow \times$
Smaller to Bigger $\rightarrow \div$

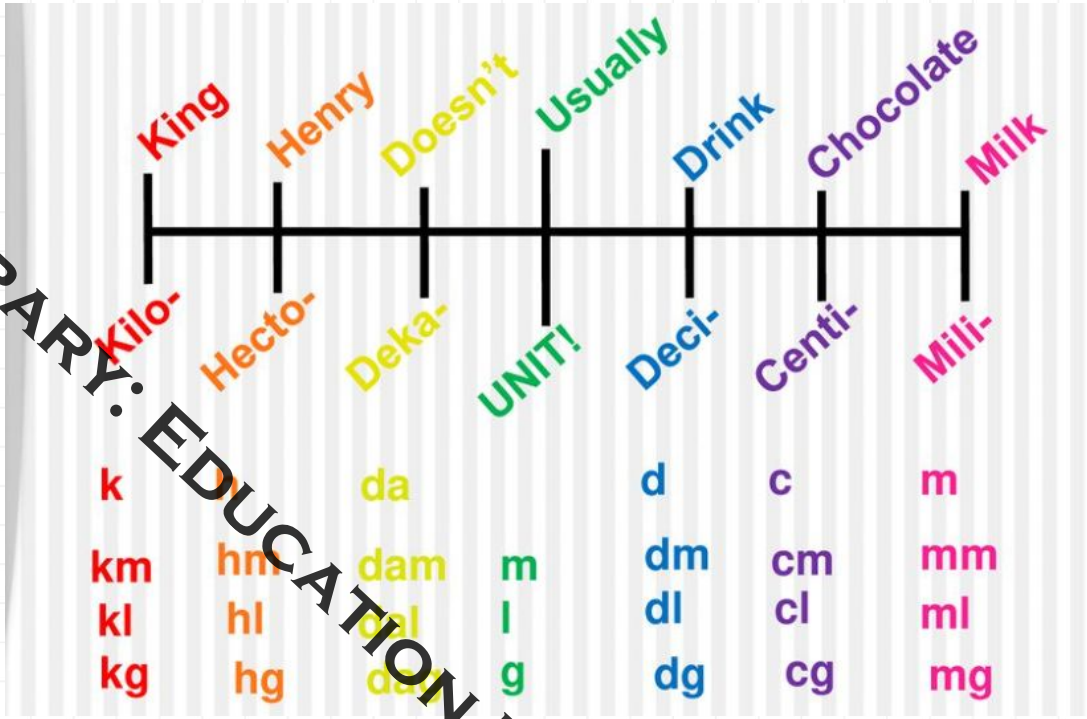
- 2) Multiply/Divide by 10 each space you moved to get to your desired unit

- 3) You found your answer

<div> <div>$\times 10$</div> <div>$\times 10$</div> <div>$\times 10$</div> <div>$\times 10$</div> <div>$\times 10$</div> <div>$\times 10$</div> </div>						
King KILO-	Henry HECTO-	Died DEKA-	(U)nusually (UNIT)	Drinking DECI-	Chocolate CENTI-	Milk MILLI-
<div> <div>$\div 10$</div> <div>$\div 10$</div> <div>$\div 10$</div> <div>$\div 10$</div> <div>$\div 10$</div> <div>$\div 10$</div> </div>						
	5 hg		-gram (weight)	?		
—	5.0 hg	50.0 dag	500.0g	5,000.0 dg	—	—
			-meter (length)		?	3.0 mm
—	—	—	—	—	0.3 cm	3.0 mm
4 kL		?	-liter (capacity)			
			—	—	—	—
kg= kilogram km= kilometer kL= kiloliter	hg= hectogram hm= hectometer hL= hectoliter	dag= dekagram dam= dekameter dal= dekaliter	g= gram m= meter L= liter	dg= decigram dm= decimeter dL= deciliter	cg= centigram cm= centimeter cL= centiliter	mg= milligram mm= millimeter mL= milliliter

- Hints... 1) Watch your decimal point move over every time you move one space.
2) The math that is used for -gram, -meter, -liter is the same.

PREFIXES IN EVERYDAY USE			
PREFIX	SYMBOL	SCIENTIFIC NOTATION	FACTOR
tera	T	10^{12}	1 000 000 000 000
giga	G	10^9	1 000 000 000
mega	M	10^6	1 000 000
kilo	k	10^3	1 000
hecto	h	10^2	100
deca	da	10^1	10
BASE UNIT	(none)	10^0	1
deci	d	10^{-1}	0.1
centi	c	10^{-2}	0.01
milli	m	10^{-3}	0.001
micro	μ	10^{-6}	0.000 001
nano	n	10^{-9}	0.000 000 001
pico	p	10^{-12}	0.000 000 000 001



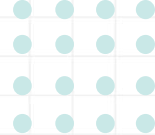
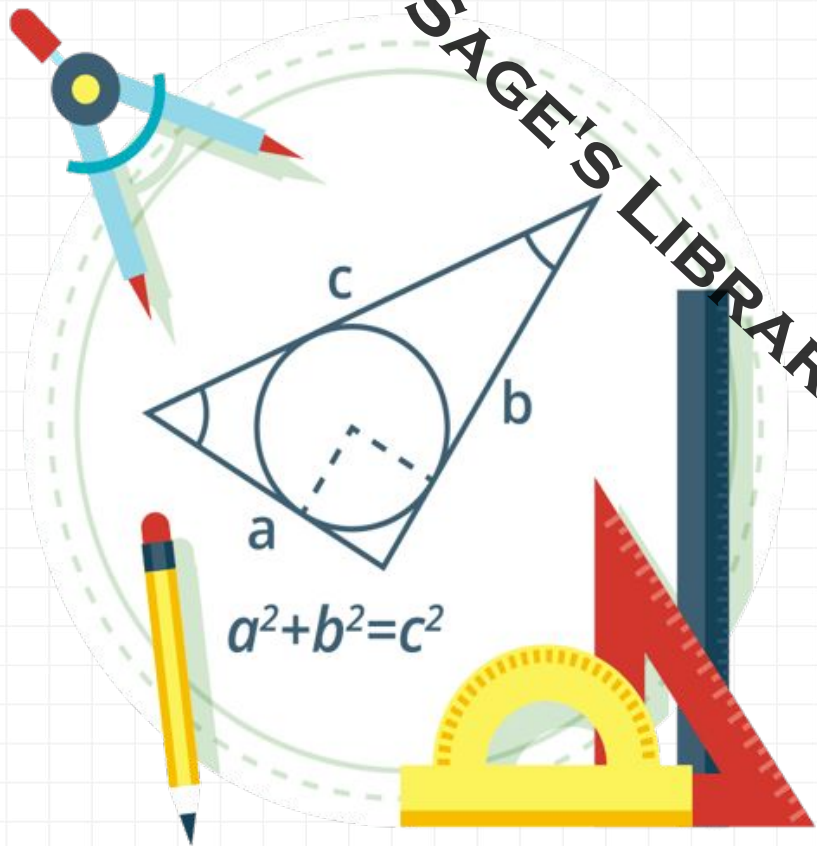
Homework:

1. 4,000L → ml
2. 10,000km → mm
3. 12.753 gr → kg
4. 30 decimetres → km
5. 4.2718 cm → km
6. 19.93 centigrams → Kg (divide, 5 spaces = 100000)
7. 0.988 Kl → ml (multiply, 6 spaces = 1000000)
8. 1348.2 km → cm (multiply, 5 spaces = 100000)
9. 2024 M → Hm (divide, 2 spaces = 100)
10. 12,978.34 centilitres → dekalitres (divide, 3 spaces = 1000)

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Geometry

Geometry is the branch of mathematics concerned with the properties and relations of points, lines, surfaces, and solids. It involves understanding shapes, angles, and the space they occupy.

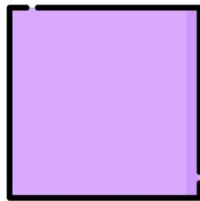


EDUCATION UNTOLED

2D Shapes vs. 3D Shapes

Examples of 2D Geometric Shapes

Square

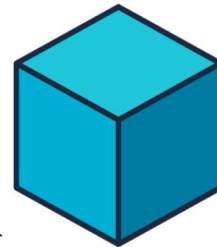


Circle

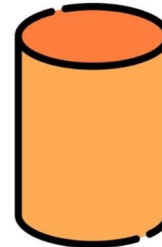


Examples of 3D Geometric Shapes

Cube



Cylinder



Surface Area

Surface Area:

- **Definition:** Surface area is the total area that the surface of a 3D object covers. It's like wrapping the object in a sheet of paper and measuring the total amount of paper you need to cover it.
- **Units:** Square units (e.g., cm^2 , m^2).
- **Calculation:** You calculate surface area by adding up the areas of all the faces or curved surfaces of the object.
 - **For a cube:** Surface Area = $6 \times \text{side}^2$ (since all faces are squares).
 - **For a cylinder:** Surface Area = $2\pi r^2 + 2\pi rh$ (includes both circular bases and the curved surface).

Examples of real-world applications:

- Painting the outside of a box.
- Wrapping a present.
- Calculating the amount of material needed to cover an object.

Volume

Volume:

- **Definition:** Volume measures the amount of space an object occupies or how much liquid or material it can hold.
- **Units:** Cubic units (e.g., cm^3 , m^3).
- **Calculation:** You calculate volume by multiplying the object's length, width, and height (or appropriate dimensions for other shapes).
 - **For a cube:** $\text{Volume} = \text{side}^3$.
 - **For a cylinder:** $\text{Volume} = \pi r^2 h$ (area of the base \times height).

Examples of real-world applications:

- Determining how much water can fill a swimming pool.
- Measuring the capacity of a container.
- Finding the amount of soil needed to fill a garden bed.

Formulas

SHAPES — perimeter (P) and area (A)

SQUARE

$$P = 4a$$

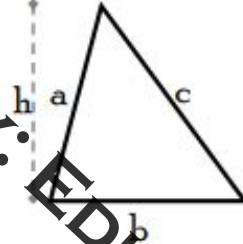
$$A = a^2$$



TRIANGLE

$$P = a + b + c$$

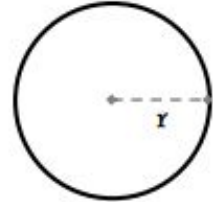
$$A = \frac{1}{2}bh$$



CIRCLE

$$C = 2\pi r$$

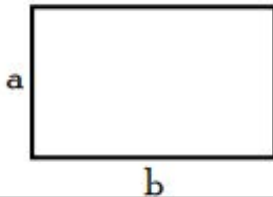
$$A = \pi r^2$$



RECTANGLE

$$P = 2a + 2b$$

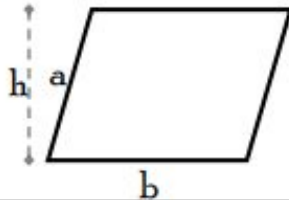
$$A = ab$$



PARALLELOGRAM

$$P = 2a + 2b$$

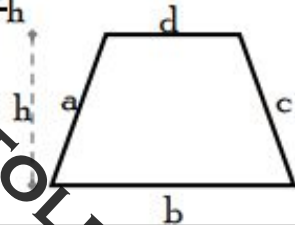
$$A = bh$$



TRAPEZOID

$$P = a + b + c + d$$

$$A = \frac{b+d}{2}h$$



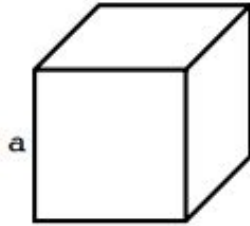
Formulas

SOLIDS — surface area (SA) and volume (V)

CUBE

$$SA = 6a^2$$

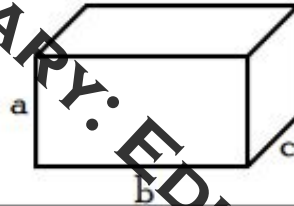
$$V = a^3$$



RECTANGULAR PRISM

$$SA = 2ab + 2ac + 2bc$$

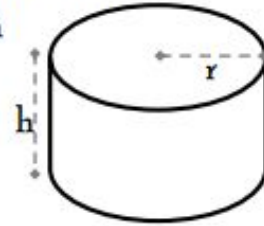
$$V = abc$$



CYLINDER

$$SA = 2\pi r(r+h)$$

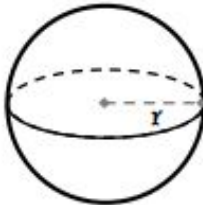
$$V = \pi r^2 h$$



SPHERE

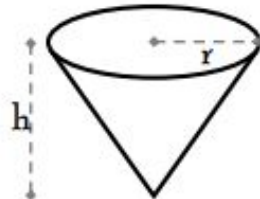
$$SA = 4\pi r^2$$

$$V = \frac{4\pi r^3}{3}$$



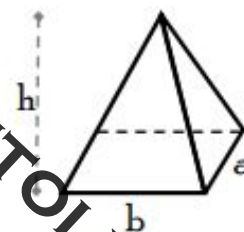
CONE

$$V = \frac{1}{3}\pi r^2 h$$



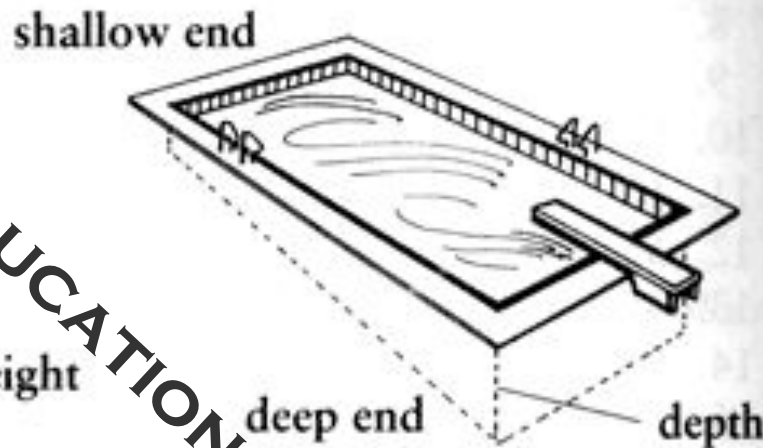
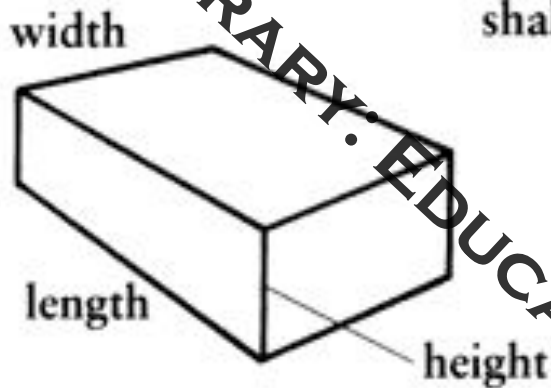
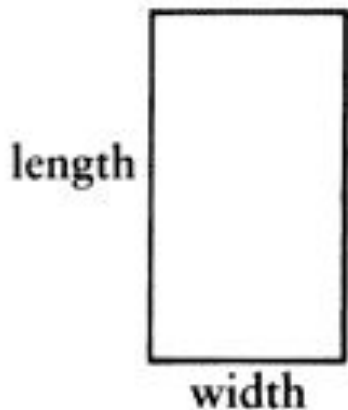
RECTANGULAR PYRAMID

$$V = \frac{1}{3}abh$$



Length vs. Width vs. Height

Size and dimension



Surface Area Practice Questions

1. **Cube:** A cube has a side length of 4 cm. What is the surface area of the cube?
2. **Rectangular Prism:** A rectangular prism has dimensions of 5 cm, 3 cm, and 7 cm. Find the surface area.
3. **Sphere:** The radius of a sphere is 6 cm. What is the surface area of the sphere?
4. **Cylinder:** A cylinder has a radius of 4 cm and a height of 10 cm. Calculate the surface area of the cylinder.
5. **Cone:** A cone has a radius of 3 cm and a slant height of 8 cm. Find the surface area of the cone.

Answers

1. **Cube:** A cube has a side length of 4 cm. What is the surface area of the cube?

Formula: Surface Area = $6 \times \text{side}^2$

Steps:

- Side length = 4 cm.
- Surface Area = $6 \times (4 \text{ cm})^2$
- Surface Area = $6 \times 16 \text{ cm}^2$
- Surface Area = 96 cm^2

Answer: The surface area of the cube is **96 cm^2** .

Answers

2. Rectangular Prism: A rectangular prism has dimensions of 5 cm, 3 cm, and 7 cm. Find the surface area.

Formula: Surface Area = $2lw + 2lh + 2wh$, where l = length, w = width, and h = height.

Steps:

- Length (l) = 5 cm, Width (w) = 3 cm, Height (h) = 7 cm.
- Surface Area = $2(5 \times 3) + 2(5 \times 7) + 2(3 \times 7)$
- Surface Area = $2(15) + 2(35) + 2(21)$
- Surface Area = $30 + 70 + 42$
- Surface Area = 142 cm^2

Answer: The surface area of the rectangular prism is **142 cm^2** .

Answers

3. Sphere: The radius of a sphere is 6 cm. What is the surface area of the sphere?

Formula: Surface Area = $4\pi r^2$

Steps:

- Radius (r) = 6 cm.
- Surface Area = $4\pi(6 \text{ cm})^2$
- Surface Area = $4\pi(36 \text{ cm}^2)$
- Surface Area $\approx 4 \times 3.14 \times 36$
- Surface Area $\approx 452.16 \text{ cm}^2$

Answer: The surface area of the sphere is approximately **452.16 cm²**.

Answers

4. Cylinder: A cylinder has a radius of 4 cm and a height of 10 cm. Calculate the surface area of the cylinder.

Formula: Surface Area = $2\pi r^2 + 2\pi rh$

Steps:

- Radius (r) = 4 cm, Height (h) = 10 cm.
- Surface Area = $2\pi(4 \text{ cm})^2 + 2\pi(4 \text{ cm})(10 \text{ cm})$
- Surface Area = $2\pi(16 \text{ cm}^2) + 2\pi(40 \text{ cm})$
- Surface Area $\approx 2 \times 3.14 \times 16 + 2 \times 3.14 \times 40$
- Surface Area $\approx 100.48 \text{ cm}^2 + 251.2 \text{ cm}^2$
- Surface Area $\approx 351.68 \text{ cm}^2$

Answer: The surface area of the cylinder is approximately **351.68 cm²**.

Answers

5. Cylinder: A cylinder has a radius of 4 cm and a height of 10 cm. Calculate the surface area of the cylinder.

Formula: Surface Area = $2\pi r^2 + 2\pi rh$

Steps:

- Radius (r) = 4 cm, Height (h) = 10 cm.
- Surface Area = $2\pi(4 \text{ cm})^2 + 2\pi(4 \text{ cm})(10 \text{ cm})$
- Surface Area = $2\pi(16 \text{ cm}^2) + 2\pi(40 \text{ cm})$
- Surface Area $\approx 2 \times 3.14 \times 16 + 2 \times 3.14 \times 40$
- Surface Area $\approx 100.48 \text{ cm}^2 + 251.2 \text{ cm}^2$
- Surface Area $\approx 351.68 \text{ cm}^2$

Answer: The surface area of the cylinder is approximately **351.68 cm²**.

Volume Practice Questions

1. **Cube:** A cube has a side length of 5 cm. What is the volume of the cube?
2. **Rectangular Prism:** A rectangular prism has dimensions of 8 cm, 6 cm, and 2 cm. Calculate its volume.
3. **Sphere:** The radius of a sphere is 7 cm. What is the volume of the sphere?
4. **Cylinder:** A cylinder has a radius of 3 cm and a height of 12 cm. What is the volume of the cylinder?
5. **Cone:** A cone has a radius of 4 cm and a height of 9 cm. Find the volume of the cone.

Answers - #1

Cube: A cube has a side length of 5 cm. What is the volume of the cube?

Formula: $\text{Volume} = \text{side}^3$

Steps:

- Side length = 5 cm.
- $\text{Volume} = (5 \text{ cm})^3$
- $\text{Volume} = 5 \times 5 \times 5$
- $\text{Volume} = 125 \text{ cm}^3$

Answer: The volume of the cube is **125 cm³**.

Answers #2

Rectangular Prism: A rectangular prism has dimensions of 8 cm, 6 cm, and 2 cm. Calculate its volume.

Formula: Volume = $l \times w \times h$

Steps:

- Length (l) = 8 cm, Width (w) = 6 cm, Height (h) = 2 cm.
- Volume = $8 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm}$
- Volume = 96 cm^3

Answer: The volume of the rectangular prism is 96 cm^3 .

Answers #3

Sphere: The radius of a sphere is 7 cm. What is the volume of the sphere?

Formula: $\text{Volume} = \frac{4}{3}\pi r^3$

Steps:

- Radius (r) = 7 cm.
- $\text{Volume} = \frac{4}{3}\pi(7 \text{ cm})^3$
- $\text{Volume} = \frac{4}{3}\pi(343 \text{ cm}^3)$
- $\text{Volume} \approx \frac{4}{3} \times 3.14 \times 343$
- $\text{Volume} \approx 1436.76 \text{ cm}^3$

Answer: The volume of the sphere is approximately **1436.76 cm³**.

Answers #4

Cylinder: A cylinder has a radius of 3 cm and a height of 12 cm. What is the volume of the cylinder?

Formula: Volume = $\pi r^2 h$

Steps:

- Radius (r) = 3 cm, Height (h) = 12 cm.
- Volume = $\pi(3 \text{ cm})^2(12 \text{ cm})$
- Volume = $3.14 \times 9 \text{ cm}^2 \times 12 \text{ cm}$
- Volume $\approx 3.14 \times 108 \text{ cm}^3$
- Volume $\approx 339.12 \text{ cm}^3$

Answer: The volume of the cylinder is approximately **339.12 cm³**.

Answers #5

Cone: A cone has a radius of 4 cm and a height of 9 cm. Find the volume of the cone.

Formula: Volume = $(1/3)\pi r^2 h$

Steps:

- Radius (r) = 4 cm, Height (h) = 9 cm.
- Volume = $(1/3)\pi(4 \text{ cm})^2(9 \text{ cm})$
- Volume = $(1/3)\pi(16 \text{ cm}^2)(9 \text{ cm})$
- Volume $\approx (1/3) \times 3.14 \times 144 \text{ cm}^3$
- Volume $\approx 150.72 \text{ cm}^3$

Answer: The volume of the cone is approximately **150.72 cm³**.

Homework Question

Mixed Shapes: A swimming pool is in the shape of a rectangular prism, with a length of 10 m, width of 4 m, and depth of 2.5 m. On top of the pool, there's a dome (hemisphere) with a radius of 4 m. Find the total volume of water the pool and dome can hold.

Homework Question – Solution

1. **Volume of the Rectangular Prism** (pool):

- Volume = $l \times w \times h$
- Volume = $10 \text{ m} \times 4 \text{ m} \times 2.5 \text{ m}$
- Volume = 100 m^3

2. **Volume of the Hemisphere** (dome):

- Volume of a sphere = $(4/3)\pi r^3$. For a hemisphere, divide by 2.
- Volume = $(1/2) \times (4/3)\pi(4 \text{ m})^3$
- Volume = $(1/2) \times (4/3)\pi(64 \text{ m}^3)$
- Volume $\approx (1/2) \times 268.08 \text{ m}^3$
- Volume $\approx 134.04 \text{ m}^3$

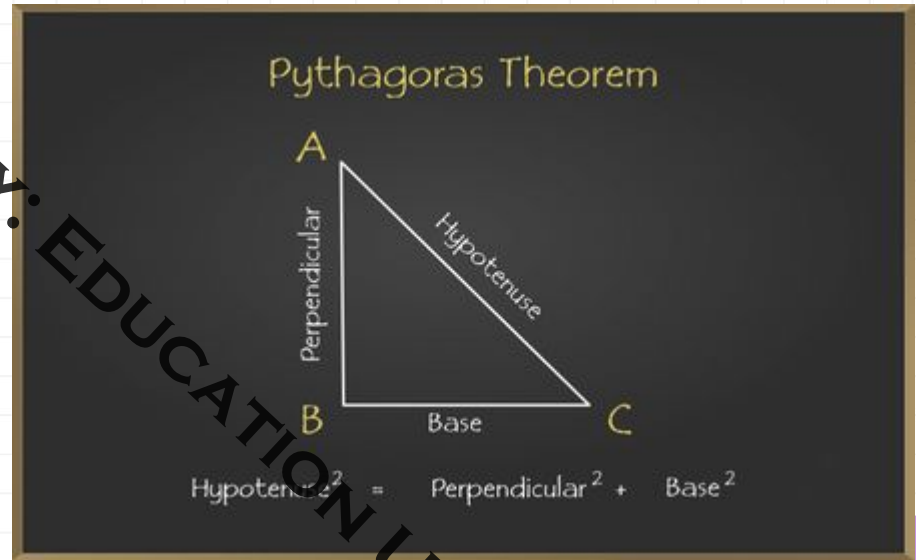
3. **Total Volume** = Volume of pool + Volume of dome

- Total Volume = $100 \text{ m}^3 + 134.04 \text{ m}^3$
- Total Volume $\approx 234.04 \text{ m}^3$

Answer: The total volume is approximately **234.04 m³**.

Triangles

Surface Area, Volume,
Pythagorean Theorem



Introduction to Triangles

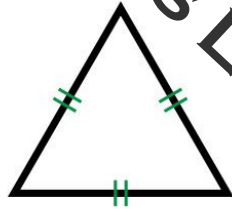
Key Points:

- A triangle is a 2D shape with three sides and three angles.
- The sum of the interior angles of a triangle always equals 180° .
- Types of triangles based on side lengths:
 - **Equilateral Triangle**: All sides and angles are equal (60° each).
 - **Isosceles Triangle**: Two sides are equal, two angles are equal.
 - **Scalene Triangle**: All sides and angles are different.

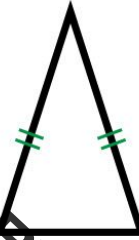
Types of triangles based on angles:

- **Acute Triangle**: All angles are less than 90° .
- **Right Triangle**: One angle is exactly 90° .
- **Obtuse Triangle**: One angle is greater than 90° .

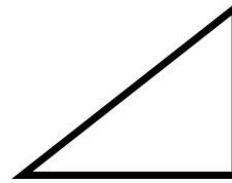
Types of Triangles



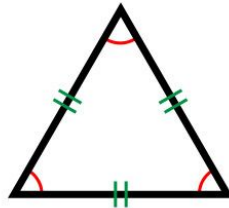
Equilateral Triangle
3 equal sides



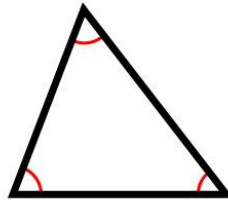
Isosceles Triangle
2 equal sides



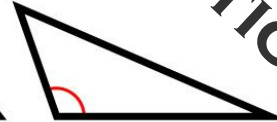
Scalene Triangle
NO equal sides



Equilateral Triangle
3 angles = 60°



Acute Triangle
3 angles $< 90^\circ$



Obtuse Triangle
1 angle $> 90^\circ$



Right Triangle
1 angle = 90°

Surface Area of Triangular Prisms

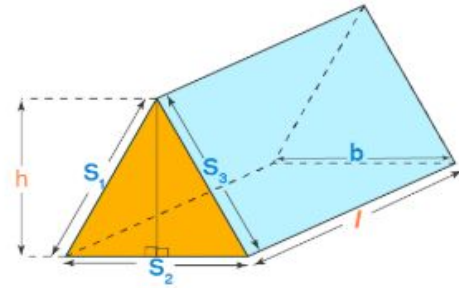
Surface area

$$= (\text{Perimeter of the base} \times \text{Length of the prism}) + (2 \times \text{Base Area})$$

$$= (S_1 + S_2 + S_3)L + bh$$

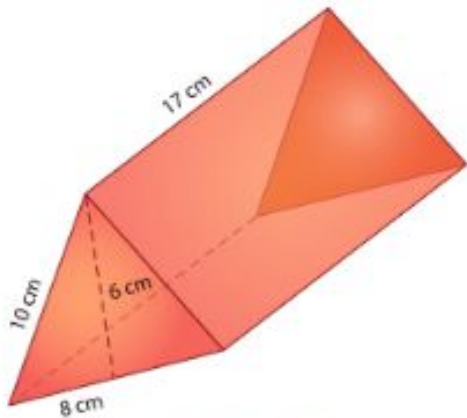
where,

- b is the bottom edge of the base triangle,
- h is the height of the base triangle,
- L is the length of the prism and
- S_1 , S_2 , and S_3 are the three edges (sides) of the base triangle
- (bh) is the combined area of the two triangular faces $[2 \times (1/2 \times bh)] = bh$



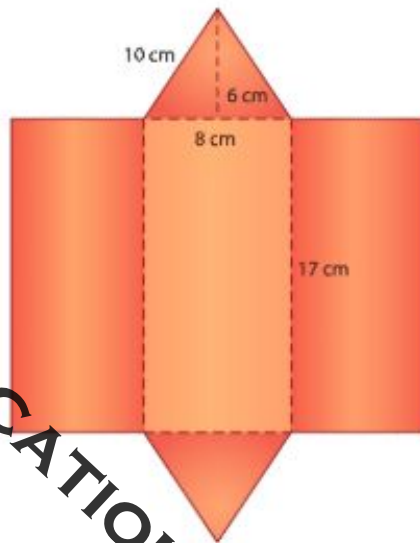
$$\begin{aligned} \text{Total Surface Area} &= (\text{Perimeter} \times \text{Length}) + (2 \times \text{Base Area}) \\ &= (S_1 + S_2 + S_3)L + bh \end{aligned}$$

What is the surface area of the figure below?



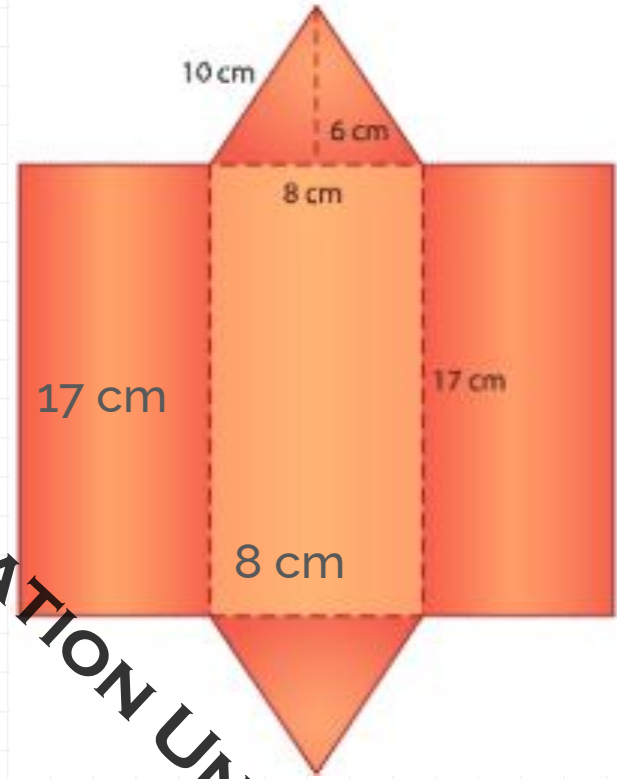
[Figure 2]

The net for this triangular prism is as follows:



Now, let's fill in the measurements for the sides of each face in order to calculate their area.

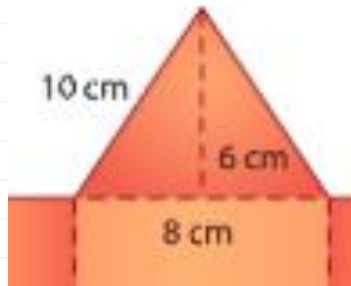
Triangular prisms have their own formula for finding surface area because they have two triangular faces **opposite** each other.



The formula

$$A = \frac{1}{2}bh$$

is used to find the area of the top and bases triangular faces, where A = area, b = base, and h = height.



The formula

$$A = lw$$

is used to find the area of the three rectangular side faces, where A = area, l = length, and w = width.



Plugging in the measurements that are given in the net, calculate the area of each face. Remember to use the correct area formula for the triangles and rectangles.

Bottom face	Top face	side	side	side
$A = \frac{1}{2}bh$	$A = \frac{1}{2}bh$	$A = S_1 \cdot H$	$A = S_2 \cdot H$	$A = S_3 \cdot H$
$\frac{1}{2}(16)(6) +$	$\frac{1}{2}(16)(6) +$	$17 \times 10 +$	$17 \times 10 +$	17×16
48 +	48 +	170 +	170 +	272
$= 708 \text{ cm}^2$				

When you add these values together, you get a surface area of 708 square centimeters for this triangular prism.

$$SA = bh + (S_1 + S_2 + S_3)H$$

Earlier, you were given a problem about the gift Max bought for his mom.

The triangular end has a base of 3 cm and height of 4 cm. The length of each side is 6 cm and the height of the prism (length of the rectangle) is 8 cm. What is the surface area of this triangular prism?

First, substitute the given values into the formula.

$$SA = bh + (s1 + s2 + s3)H$$

$$SA = 3(4) + (3 + 6 + 6)(8)$$

Next, multiply the base times height for the area of the triangles (bh), and add their three sides (s1 + s2 + s3).

$$SA = 12 + 15(8)$$

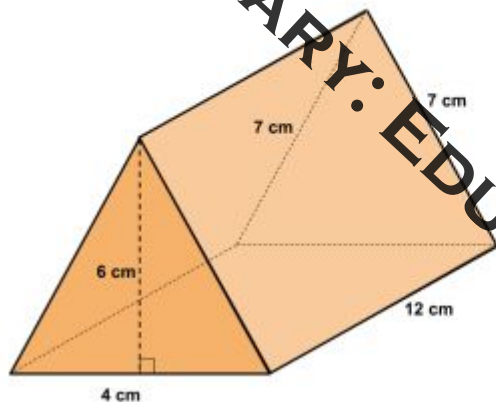
Then, multiply the sum of the triangle sides by the height of the prism (H) and add the values together for the answer, remembering to include the appropriate unit of measurement.

$$SA = 12 + 120$$

$$SA = 132 \text{ cm}^2$$

Try It!

Calculate the surface area of the following triangular prism:



Solution

First, substitute the given values into the formula.

$$SA = bh + (s1 + s2 + s3)H$$

$$SA = 4(6) + (4 + 7 + 7)(12)$$

Next, multiply the base times height for the area of the triangles (bh) and add the three sides of the triangle (s1 + s2 + s3).

$$SA = 24 + 18(12)$$

Then, multiply the sum of the triangle sides by the height of the prism (H), and add the values together for the answer, making sure to include the appropriate unit of measurement.

$$SA = 24 + 216$$

$$SA = 240 \text{ cm}^2$$

Volume of a Triangular Prism

The volume of a triangular prism is the amount of space it takes up in three-dimensional space. To find the volume, we multiply the area of the triangular base by the length of the prism. Since the base is a triangle, its area is found using the formula for the **area of a triangle**.

The formula for the area of a triangle is:

Area = $\frac{1}{2} \times \text{base} \times \text{height}$

This simple formula allows for easy calculation of the area of any triangle, providing a foundational concept in geometry and mathematics.

Volume of a Triangular Prism

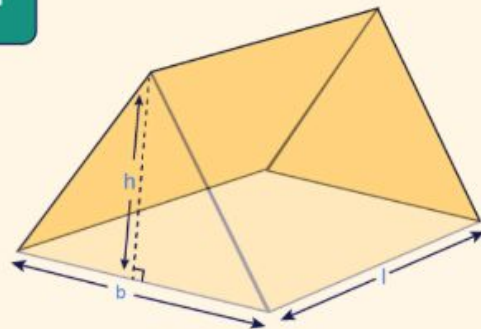
$$\text{Volume (V)} = \frac{1}{2} b \times h \times l$$

here,

b = base length

h = base height

l = length of the prism



Example #1

Find the volume of a triangular prism with a base length of 8 units, a height of 5 units, and a length of 12 units.

Solution

Solution:

To find the volume of the triangular prism, we use the formula $V = \frac{1}{2} \times b \times h \times l$, where b is the base length, h is the height, and l is the length:

$$V = \frac{1}{2} \times 8 \times 5 \times 12$$

$$V = \frac{1}{2} \times 8 \times 5 \times 12$$

$$V = 240 \text{ cubic units}$$

Therefore, the volume of the triangular prism is 240 cubic units.

Example 2

Calculate the surface area of a triangular prism with a base area of 15 square units, a length of 8 units, and a perimeter of the base as 20 units.

Solution

We compute the surface area of the prism using the formula:

$$\text{Surface area} = (\text{Perimeter of the base} \times \text{Length}) + (2 \times \text{Base Area})$$

$$\text{Surface area} = (20 \times 8) + (2 \times 15)$$

$$\text{Surface area} = 160 + 30$$

$$\text{Surface area} = 190 \text{ square units}$$

Thus, the surface area of the triangular prism is 190 square units.

Example 3

Find the volume of a triangular prism with a base area of 35 cm^2 and a length of 12 cm.

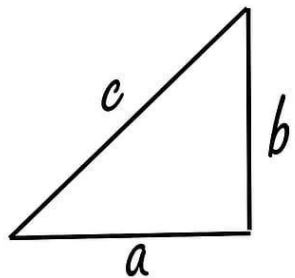
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Solution

Using the formula $\text{Volume} = \text{base area} \times l$, where b is the base, h is the height, and l is the length:

$$\text{Volume} = 35 \times 12$$

$$\text{Volume} = 420 \text{ cubic cm}$$



$$a^2 + b^2 = c^2$$

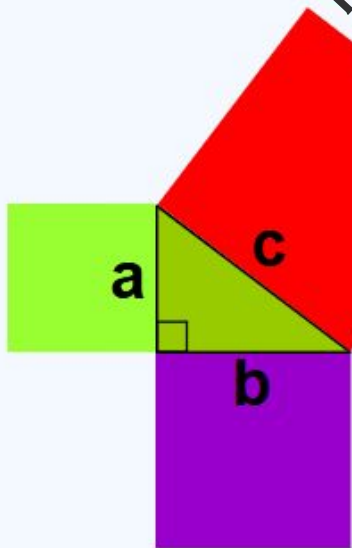


Pythagorean Theorem

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It is called "Pythagoras' Theorem" and can be written in one short equation:



$$a^2 + b^2 = c^2$$

A visual representation of the equation $a^2 + b^2 = c^2$. It shows a green square labeled a^2 , a purple square labeled b^2 , and a red square labeled c^2 , connected by a plus sign and an equals sign.

Note:

- **c** is the **longest side** of the triangle
- **a** and **b** are the other two sides

Definition

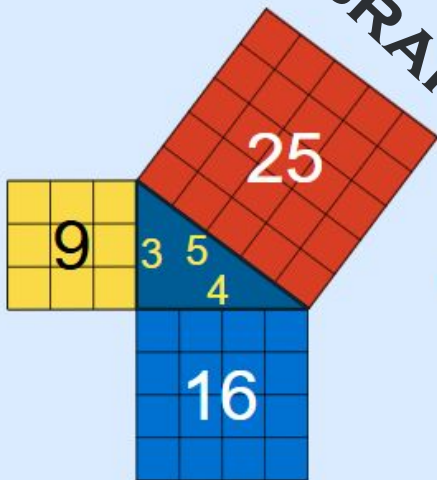
The longest side of the triangle is called the "hypotenuse", so the formal definition is:

In a right angled triangle:
the square of the hypotenuse is equal to
the sum of the squares of the other two sides.

Sure ... ?

Let's see if it really works using an example.

Example: A "3, 4, 5" triangle has a right angle in it.



Let's check if the areas **are** the same:

$$3^2 + 4^2 = 5^2$$

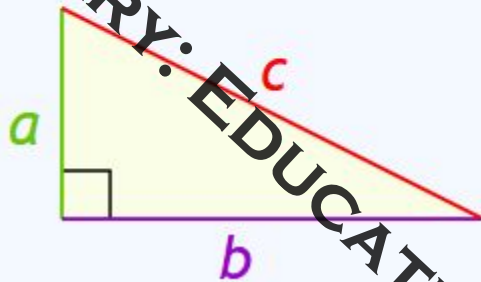
Calculating this becomes:

$$9 + 16 = 25$$

It works ... like Magic!

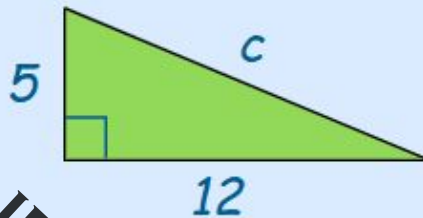
How Do I Use it?

Write it down as an equation.



$$a^2 + b^2 = c^2$$

Example: Solve this triangle



Start with: $a^2 + b^2 = c^2$

Put in what we know: $5^2 + 12^2 = c^2$

Calculate squares: $25 + 144 = c^2$

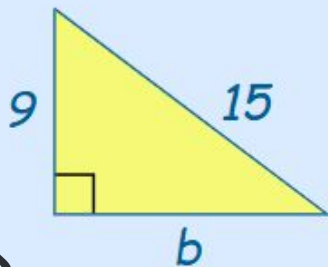
$25 + 144 = 169$: $169 = c^2$

Swap sides: $c^2 = 169$

Square root of both sides: $c = \sqrt{169}$

Calculate: $c = 13$

Example: Solve this triangle.



Start with: $a^2 + b^2 = c^2$

Put in what we know: $9^2 + b^2 = 15^2$

Calculate squares: $81 + b^2 = 225$

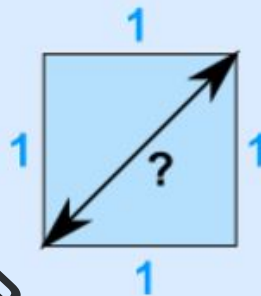
Take 81 from both sides: $81 - 81 + b^2 = 225 - 81$

Calculate: $b^2 = 144$

Square root of both sides: $b = \sqrt{144}$

Calculate: **$b = 12$**

Example: What is the diagonal distance across a square of size 1?



Start with: $a^2 + b^2 = c^2$

Put in what we know: $1^2 + 1^2 = c^2$

Calculate squares: $1 + 1 = c^2$

$1 + 1 = 2$: $2 = c^2$

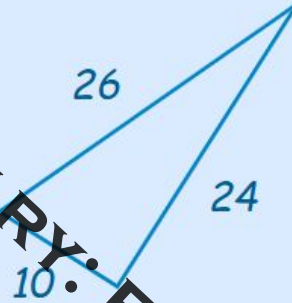
Swap sides: $c^2 = 2$

Square root of both sides: $c = \sqrt{2}$

Which is about: $c = 1.4142...$

It works the other way around, too: when the three sides of a triangle make $a^2 + b^2 = c^2$, then the triangle is right angled.

Example: Does this triangle have a Right Angle?



Does $a^2 + b^2 = c^2$?

- $a^2 + b^2 = 10^2 + 24^2 = 100 + 576 = \mathbf{676}$
- $c^2 = 26^2 = \mathbf{676}$

They are equal, so ...

Yes, it does have a Right Angle!

Example: Does an 8, 15, 16 triangle have a Right Angle?

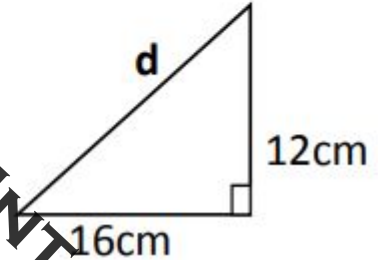
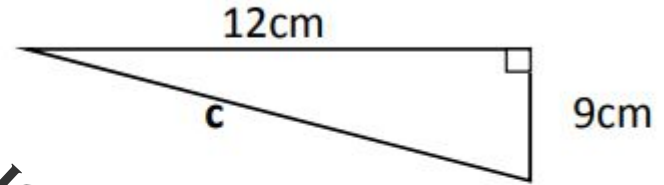
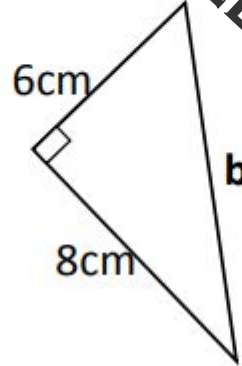
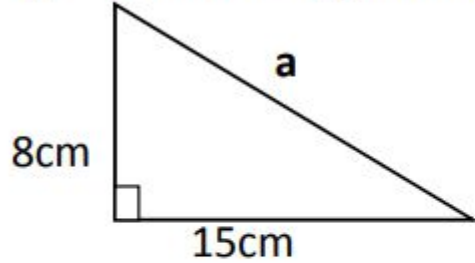
Does $8^2 + 15^2 = 16^2$?

- $8^2 + 15^2 = 64 + 225 = 289$,
- but $16^2 = 256$

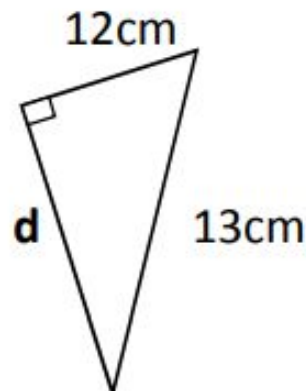
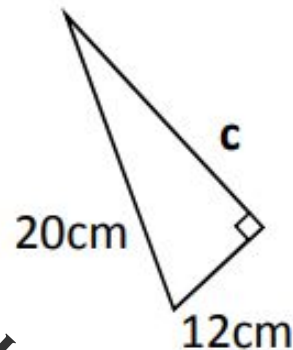
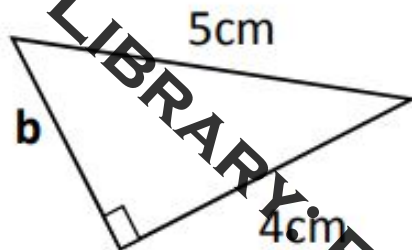
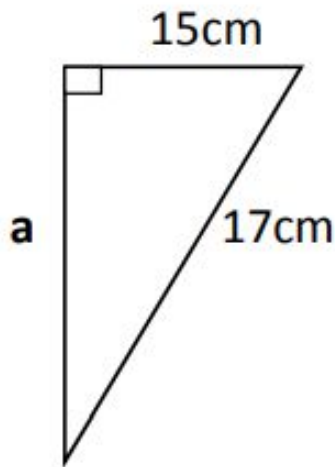
So, NO, it does not have a Right Angle

Homework

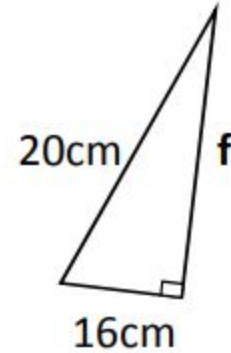
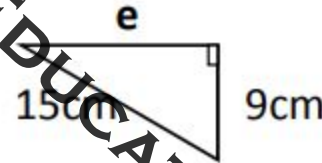
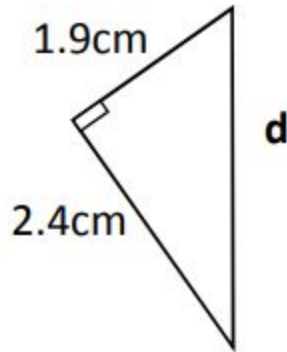
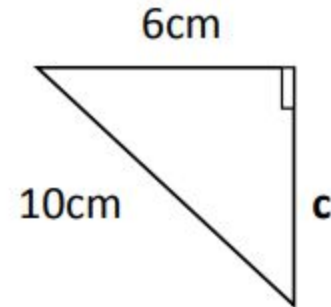
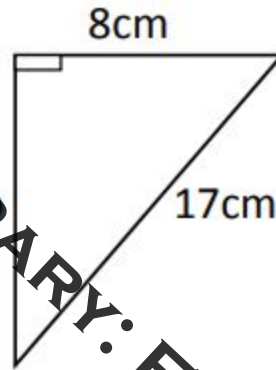
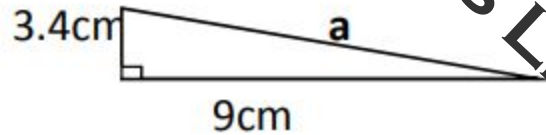
- 1) Find the hypotenuse of the following triangles.



2) Using Pythagoras find the lengths of the sides labelled with letters.



- 3) Find the missing lengths of the triangles below. If necessary, round answers to 1 decimal place.



Answers:

1) $a = 17\text{cm}$ $b = 10\text{cm}$ $c = 15\text{cm}$ $d = 20\text{cm}$

2) $a = 8\text{cm}$ $b = 3\text{cm}$ $c = 16\text{cm}$ $d = 5\text{cm}$

3) $a = 9.6\text{cm}$ $b = 15\text{cm}$ $c = 8\text{cm}$ $d = 3.1\text{cm}$ $e = 12\text{cm}$ $f = 12\text{cm}$

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